Differential equation models, whether ordinary, delay, partial or stochastic, imply a continuous overlap of generations. There are some species which have no overlap whatsoever between successive generations and so population growth is in discrete steps. In a discrete model, it is relatively easy to understand how different factors contribute to change in population size. However, in many biological situations, modeling a population to reproduce and die synchronously in each generation is not realistic and can lead to misleading results. A discrete model is easy to implement in a spreadsheet program, but mathematically manipulating the equation is cumbersome.

**Simple Models:** Let and  denote the population at time  and  respectively. The discrete time population model is of the form

 

where  is a nonlinear function of .

Suppose that the population one step later is simply proportional to the current population, that is . Then from (1) we have

 

For  equation (2) implies









Finally we get,  

Here is the net reproductive rate and  is the starting value of the population. If  then population grows geometrically and if  then population decays geometrically. For most populations or for long times this simple model is not very realistic.

**Q-01:** Describe the single species discrete time population model.

**Discrete Malthusian model:** In discrete Malthusian model, it is assume that the number of births and the number of deaths in any generation is proportional to the population size. Let and  denote the population at time  and  respectively. Then





, where  

where  is called intrinsic growth parameter of the population.

Taking , we get the following recursion relation





Similarly, 





Then  that is .

From the above model we see that,









.

This is known as Malthusian growth law or exponential growth law. The solution of the model is known as exponential function.

Behavior of the solution:

Case-I: For , . This is the case of extinction.

Case-II: For , . This is the case of unlimited growth.

Case-III: For , . This is the case of constant population.

**Q-02:** Describe the Malthusian model for single species discrete time.

**Discrete logistic model:** The continuous Verhulst logistic model is

 

where, is called the Logistic model or Pearl-Verhulst model. Here  is called carrying capacity,  and  are positive constants and  is very small compared to .

Let and  denote the population at time  and  respectively.

Thenthe logistic model for discrete time is

 

which is analogue of the continuous logistic growth model but is not at all, because the steady state is not . The solution and their dependence on the parameter  are very different. A drawback of this model is that  then . To derive it from the continuous Verhulst equation we replace the derivative  with a difference from with time step 1 to obtain





 

which is the Verhulst logistic model for single species discrete time.

For large  ther should be a reduction in the growth rate to make it a more realistic model but  should remain nonnegative. One such model, known as the Ricker curve is

 

Here  for all  if .

**Q-03:** Describe the Verhulst logistic model for single species discrete time.

**Discrete Delay model:**

**Problems**

**Problem-01:** A population grows accordingly to the equation , where  and  are positive constants. Find the stability of the equilibrium population. Determine the equilibrium population size.

**Answer:** We have



where and  are positive constants.

For equilibrium population we have















Hence the equilibrium population is .

For linearization about  we put  with  in (1). Thus we have







Since  so neglecting higher order terms, we get





This shows that as . Thus the equilibrium  i.e. is stable.

**Problem-02:** A population grows accordingly to the equation , where  and  are positive constants. Determine the equilibrium state and its stability for this population.

**Answer:** We have

 

where and  are positive constants.

For equilibrium state we have

















Hence the equilibrium state is .

**Stability:** For stability we linearize (1) by putting  with  . Thus we have













Since  so neglecting higher order terms, we get





This shows that as . Thus the equilibrium  i.e.  is stable.

**Problem-03:** A population grows accordingly to the equation , where  and  are positive constants. Determine the equilibrium state and its stability for this population.

**Answer:** We have

 

where and  are positive constants.

For equilibrium state we have













Hence the equilibrium state is .

**Stability:** For stability we linearize (1) by putting  with  . Thus we have







Since  so neglecting higher order terms, we get





This shows that as . Thus the equilibrium  i.e.  is stable.